IB MYP YEAR 5

YEAR 10 Extended
Mathematics

Assessment #2

TRANSFORMATION OF FUNCTIONS

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Teacher: **Ms. Li, Mr. So & Mr. Wong**

Date of task: **Friday, November 2, 2012** Time allowed: **60 minutes**

Student's **Performance** in Different **Criterion**			
B	6	**C**	6

INSTRUCTIONS:

- Read the instructions for all questions carefully.
- Show all work, steps and proper units.
- Ask the teacher for scrap paper, but any work on the scrap paper will **NOT** be marked.
- Write in **PENCIL**.
- **GDC** is allowed.
- Allowed to use **non-electronic dictionary**.

ASSESSMENT:

- Read the criteria descriptors carefully before you start your work. This will give you a clear understanding of what is required and what a quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest level in this task.
- This task assesses Criteria **B & C** considering ALL the questions.
  - Criterion **C** will be assessed as an **overall impression** on the presentation of work in this assessment.

Criterion B: INVESTIGATING PATTERNS

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	
1–2 Do Maths	The student is able to • Find **some** of the equations of the curves in **part A**. • Recognize and describe **some** of the transformations in **part A**.	The student **applies, with some guidance**, mathematical problem-solving techniques to recognize **simple** patterns.	**(0-8)**
3–4 General Rule	The student is able to • Do **most** of the questions in **Part A** and **some** questions in **Part B**. • Apply knowledge to unfamiliar situations in **Q10** and **Q11**.	The student • **selects and applies** mathematical problem-solving techniques to recognize patterns, and • **suggests** relationships or general rules.	**Teacher's Final Grade**
5–6 Test it	The student is able to • Fulfill the requirements above. • Deduce a general form for **Q12**. • Apply and justify the general from in **Q13**. • Find another transformation that works in **Q14**.	The student • **selects and applies** mathematical problem-solving techniques to recognize patterns, • **describes** them as relationships or general rules, and • **draws conclusions** consistent with findings.	**(0-8)**
7–8 Prove it	The student is able to • Fulfill the requirements above • Prove mathematically the answer in **Q15**. • Identify the unfamiliar transformation in **Q16** and find its equation in **Q17**.	The student • **selects and applies** mathematical problem-solving techniques to recognize patterns, • **describes** them as relationships or general rules, • **draws conclusions** consistent with findings, and • **provides justifications or proofs**.	

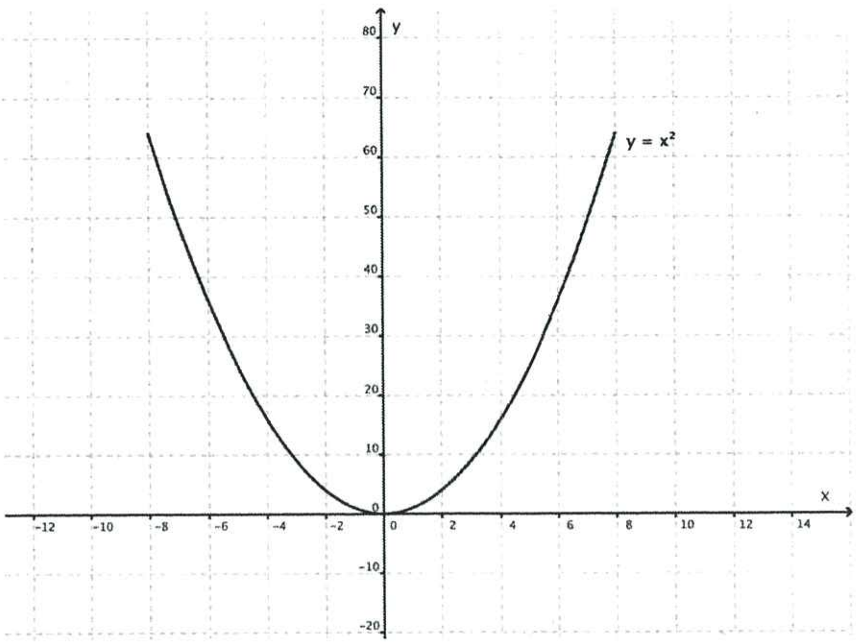
Criterion C: COMMUNICATION IN MATHEMATICS

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	
1–2	There are **some errors** or inconsistencies in use of **terminology**. There are **some** errors in the writing of **equations.** Narrative is **difficult to follow**.	• The student shows **basic** use of mathematical language **and/or** forms of mathematical representation. • The lines of reasoning are **difficult to follow**.	(0-6)
3–4	Generally students use **correct terminology** accurately, with only a **few errors.** Equations are **mostly** written **clearly** and accurately. Narrative can be **followed**, and diagrams are **clear** and labeled.	• The student shows **sufficient** use of mathematical language **and** forms of mathematical representation. • The lines of reasoning are **clear** though not always **logical** or **complete**. • The student moves between different forms of representation **with some success**.	**Teacher's Final Grade** (0-6)
5–6	The student uses the correct terminology accurately for **most of the problems.** Equations are **clear** and **accurate** Narratives are **concise**, **logical** and **complete**. **All** diagrams are **clear** and labeled	• The student shows **good** use of mathematical language **and** forms of mathematical representation. • The lines of reasoning are **concise**, **logical** and **complete**. • The student moves **effectively** between different forms of representation.	

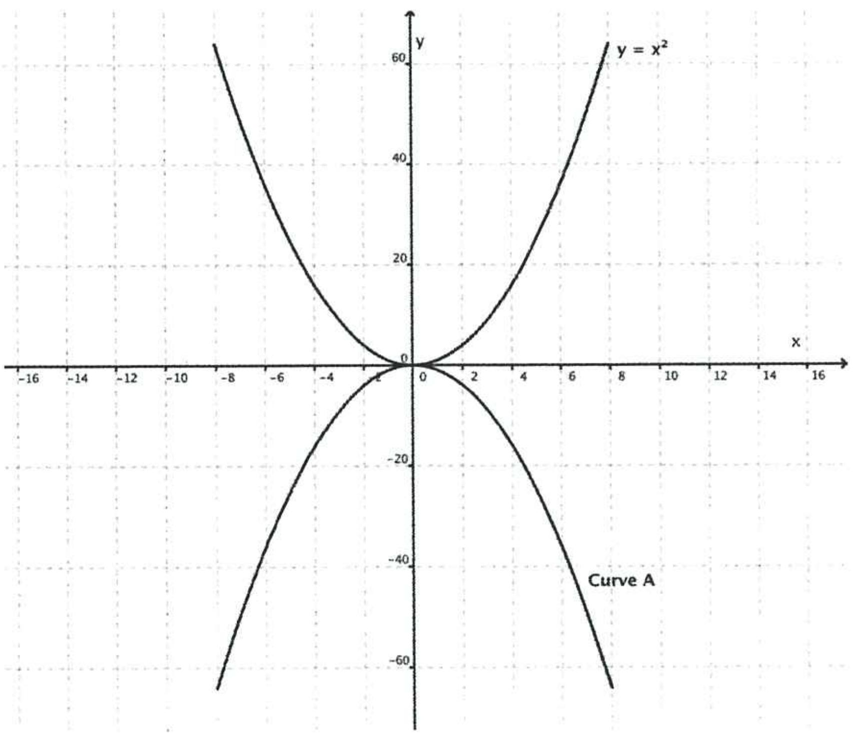
PART A (Suggested time: 20 minutes)

When the designers of the Hong Kong MTR were creating the now-famous logo, they decided to use transformations of functions.

They started with a parabola as in the graph below:



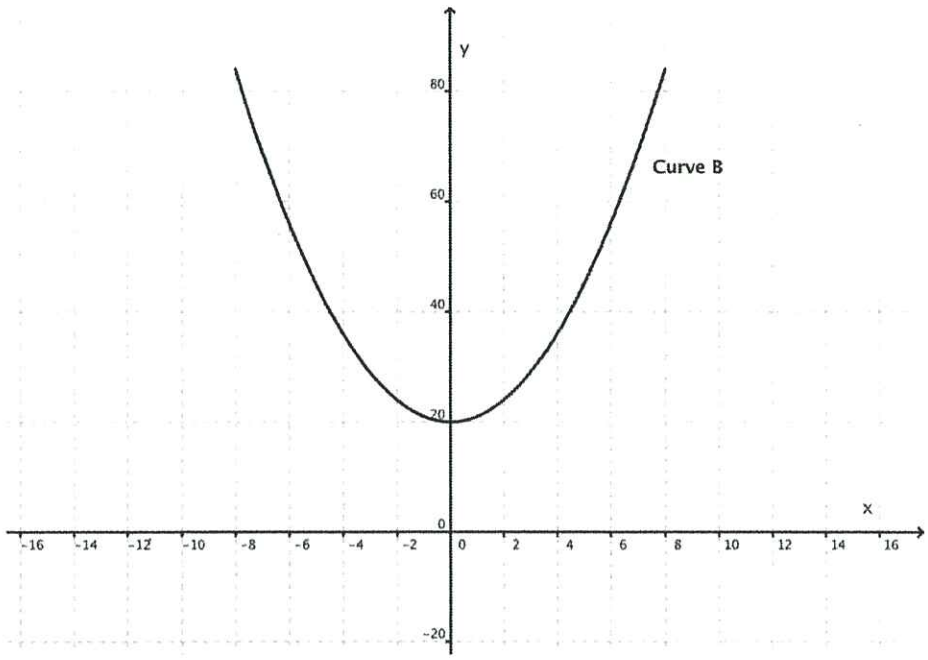
They then reflected this curve by the x-axis:



1. If the **starting curve** has the equation $\mathbf{y = x^2}$, what is the **equation of curve A**?

Let $f(x) = x^2$ and curve A be $g(x)$
$\because g(x)$ is a reflection of $f(x)$ along x-axis,
$\therefore g(x) = -f(x) = -x^2$
$\therefore$ The equation of curve A is $y = -x^2$.

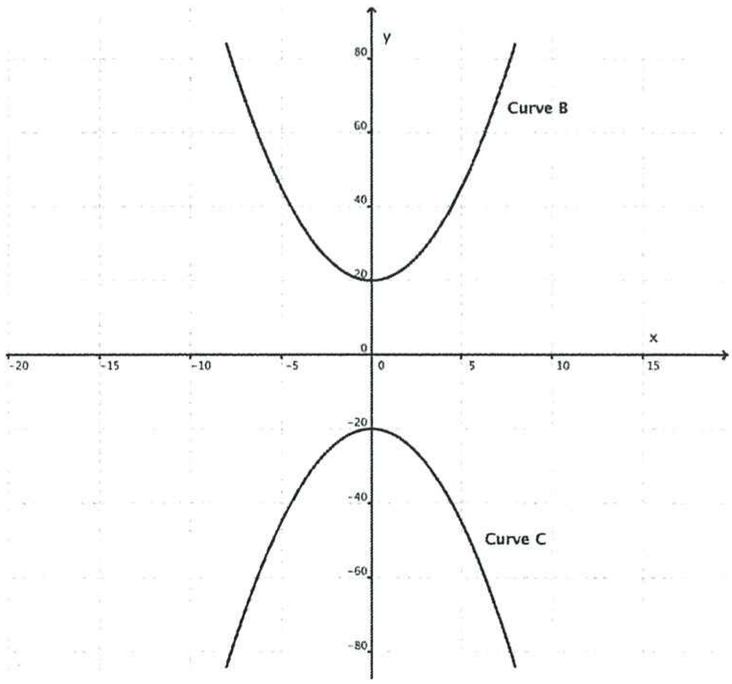
The designers then performed a transformation from their **starting curve**. It became this:



2. What do you think the **equation of curve B** is?

Let $f(x) = x^2$ and curve B be $g(x)$.
$\because$ $g(x)$ is translated 20 units upward along y-axis, $\rightarrow$ $g(x) = f(x) + 20 = x^2 + 20$
$\therefore$ Equation of curve B is $x^2 + 20$.

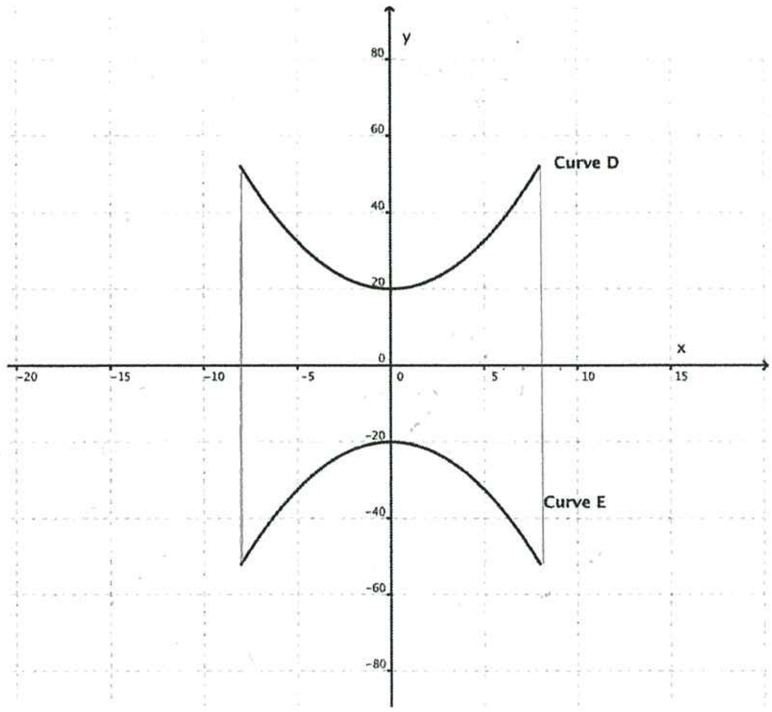
After the designers **reflected curve B** by the **x-axis**, they got **curve C**:



3. What do you think the **equation of curve C** is?

Let $f(x) = x^2 + 20$ and curve C be $g(x)$
$\because$ $g(x)$ is a reflection of $f(x)$ along x-axis, $\rightarrow$ $g(x) = -f(x) = -(x^2+20) = -x^2 - 20$
$\therefore$ Equation of curve C is $-x^2 - 20$.

The designers then went on to make one further alteration. The diagram below shows the results of that alteration. (Curves D and E are sketched on the same axes as B and C were on the last graph):



4. Describe in words what this last alteration was (to make **curve B become curve D** and **C become E**)?

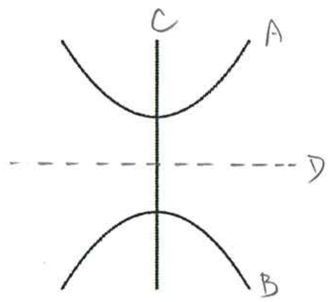
The alternation as a result was to limit the maximum value of curve B and minimum value of curve C. For a positive parabola, the curve can be extended to infinity, while a negative parabola has the same characteristics too. In order to limit the curve to stretch to infinity, the designers create a range for the x or y-axis so that the curve B is limited to its maximum value to form curve D, while curve C is limited to its minimum value to form curve E. By creating a range, the curve will be unable to stretch beyond the limit.

5. What do you think the **equation of curve D** is?

$\because$ Curve D is on the same axes as B, $\therefore$ Equation of curve B is $y = x^2 + 20$. → However, because of the limit, $\therefore$ Equation of curve D is $y = x^2 + 20$ $(-8 \leq x \leq 8)$

6. What do you think the **equation of curve E** is?

$\because$ Curve E is on the same axes as C, $\therefore$ Equation of curve E is $y = -x^2 - 20$ → However, because of the limit, $\therefore$ Equation of curve E is $y = -x^2 - 20$ $(-8 \leq x \leq 8)$

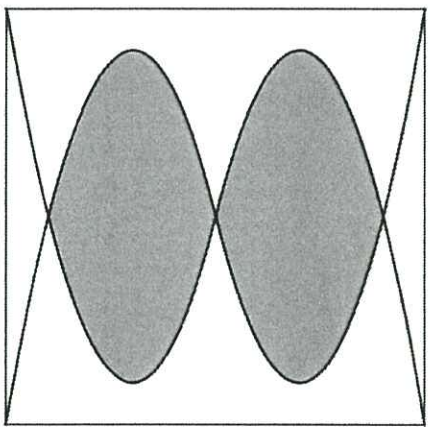


7. The "finished" logo is on the left. Describe fully the symmetry of the design

There are 4 functions in this "finished" logo ~~that~~ correlates to the symmetry By rotation ~~in~~ the diagram, there are marked as A, B, C, and D respectively. The symmetry of the design is the transformation between A and B. With D, a line with a value of y, is stretched to the midpoint of the vertices of A and B, which allows the reflection of A and B to or from each other. This means the image above D and below are they same, or symmetrical. Also, C is the line of symmetry for the curves A and B. Since they are parabolas, they are symmetrical when it is cut through the vertices. However, it is the nature that a parabola is symmetrical, and not C allowing A and B to be symmetrical.

PART B (Suggested time: 40 minutes)

Here is a logo of a well-known sportswear company:

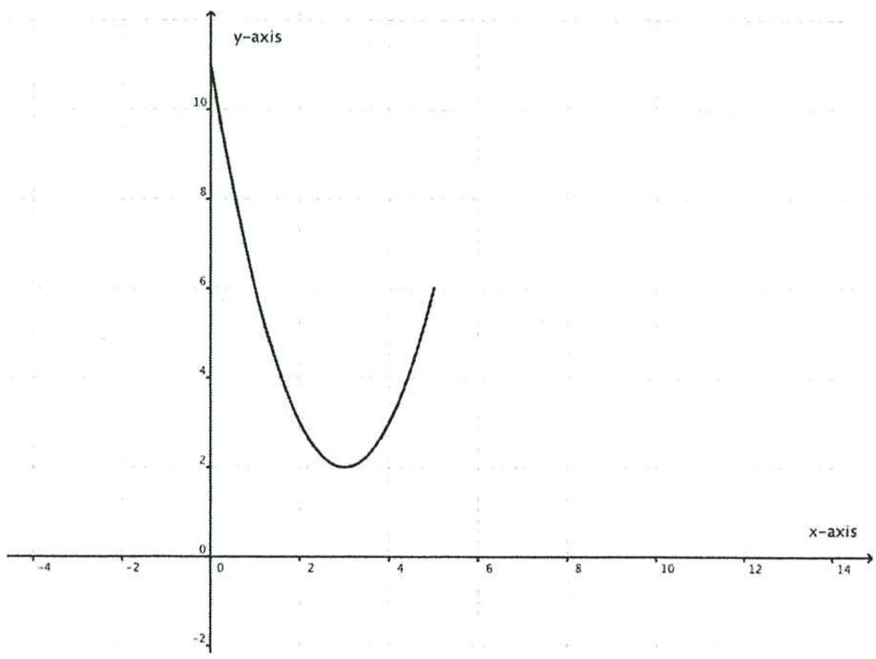


Two students, Ahmed and Delinda, are interested in the mathematics of the curves that make up the logo, and the relationship between the curves.

Ahmed believes that the design is really based on a simple curve (a quadratic), which is then transformed using simple transformations.

He says that the "basic unit" of the design below is

$$y = x^2 - 6x + 11 \qquad 0 \leq x \leq 5$$



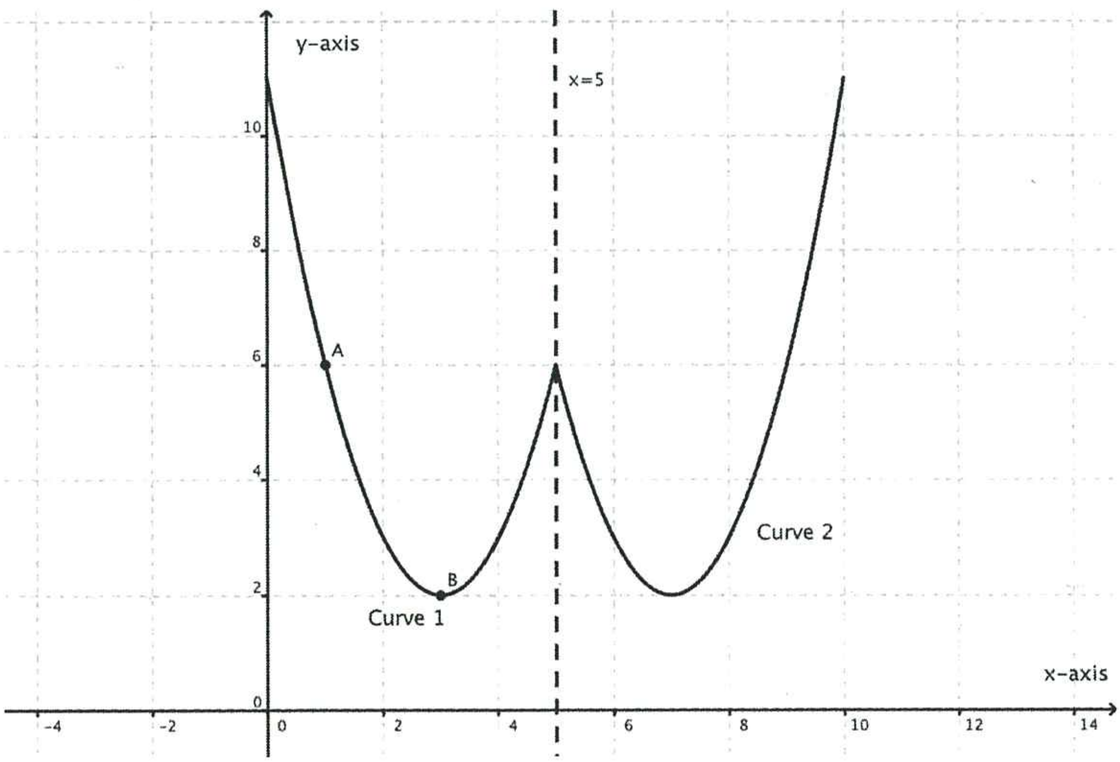
8. Show that the equation **$y = x^2 - 6x + 11$** can be written as the **second form $y = (x - 3)^2 + 2$**.

By using 'completing the square',
$y = x^2 - 6x + 11$
$= \left[x^2 - 6x + \left(\frac{6}{2}\right)^2\right] - \left(\frac{6}{2}\right)^2 + 11$
$= (x^2 - 6x + 3^2) - 3^2 + 11$
$y = (x^2 - 6x + 9) - 9 + 11$
$= (x-3)^2 + 2$

9. Describe how this second form of the equation matches up with key features of the graph?

By the formula $y = a(x-h)^2 + k$ when (h, k) are the vertex of the function, $y = (x-3)^2 + 2$ shows the vertex is $(3, 2)$, and because $a = 1 > 0$, it suggests the function is a positive curve that it is facing upwards. Thus, the information of the vertex and nature of the curve indicates the vertex $(3, 2)$ is the minimum value of the curve, while the maximum value stretches to infinity. Moreover, $y = (x-3)^2 + 2$ suggests a key feature of transformation. When $f(x) = x^2$, $y = (x-3)^2 + 2 = g(x) = f(x-3) + 2$, which represents this function is translated 3 units to the right along x-axis, and 2 units upwards along y-axis from $f(x) = x^2$. However, this key feature is only valid if $f(x) = x^2$, or the most basic function is quadratics.

He goes on to say that this curve is **reflected in the line x = 5** as shown below:



10. A is the point (1, 6). What are the coordinates of the image of A after reflection in **x = 5**?

$\because$ A is 4 units far from $x=5$ on the x-axis,
Its reflection of x-coordinates should also be 4 units away from $x=5$,
$\therefore$ The x-coordinates of the reflection is $1+4+4=9$.

$\because$ The line stretched from the y-coordinates of A is perpendicular to $x=5$,
Its reflection of y-coordinates should remain unchanged.
$\therefore$ The y-coordinates of the reflection is 6.

$\therefore$ The coordinates of the image of A after reflection in $x=5$ is $(9, 6)$.

11. B is the point (3, 2). What are the coordinates of the image of B after reflection in **x = 5**?

∵ B is 2 units away from $x=5$ on x-axis, Its reflection on the x-coordinates should also be 2 units away from $x=5$.
∵ The line stretched from y-coordinates of A is perpendicular to $x=5$, Its reflection of y-coordinates remain unchanged.

∴ The x-coordinates of the reflection is $3+2+2=7$. ∴ The y-coordinates of the reflection is 6.
∴ The coordinates of the image of B after reflection in $x=5$ is $(7,6)$.

12. P is the general point (x, y). What are the coordinates of the image of P after reflection in **x = 5**?

x-coordinates: $x+2(5-x) = x+10-2x = -x+10$
y-coordinates: y
∴ The coordinates of the image P after reflection in $x=5$ is $(-x+10, y)$

13. Ahmed takes the x-coordinate of his answer to Q12, and substitutes this in for x in the original equation $y = x^2 - 6x + 11$. He (correctly) believes that it gives him the equation of curve 2. What equation for curve 2 does Ahmed get, and what range of values of x does it apply to?

$y = (-x+10)^2 - 6(-x+10) + 11$
$= x^2 - 20x + 100 + 6x - 60 + 11 \rightarrow y = x^2 + 26x + 51$
∴ Equation of curve 2 is $y = x^2 - 14x + 55$ $(5 \le x \le 10)$

Delinda believes that Curve 1 in the diagram above can be transformed into Curve 2 **by a different way**.

14. What transformation(s) is/are Delinda thinking of? Give as many details as possible.

The transformation Delinda is thinking of involves reflection and translating, while Ahmed's way only involves reflection. Delinda's way is to reflect curve A along y-axis, then translate the function to the right to the position curve 2 is at. The advantage of this is it can apply the equation of $f(-x)$ for the reflection along y-axis, rather than drawing the line of symmetry at $x=5$, and apply the equation of $f(x-a)$ for translating the function to the right along x-axis.

by how many units!

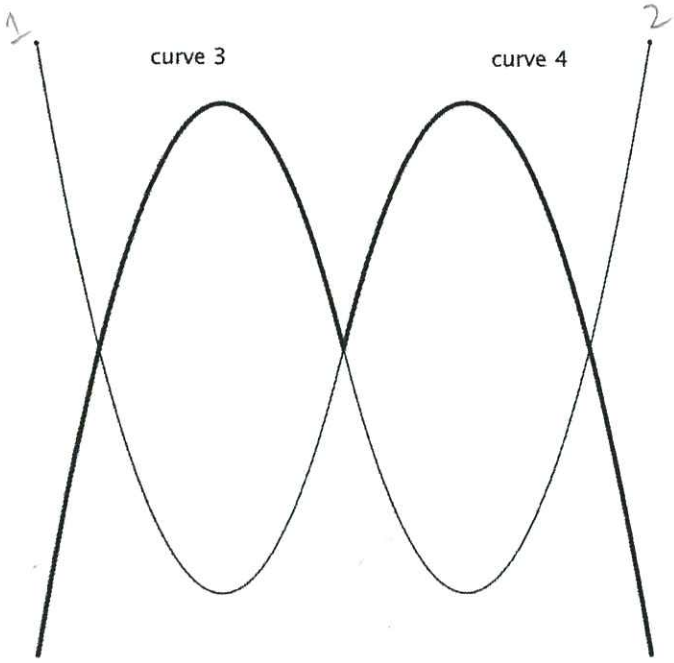
15. Is it possible for both Ahmed and Delinda to be right? Explain your answer.

Ahmed and Delinda can both be correct. Below are the mathematical steps for Delinda's method:

Calculations

Let equation of curve 1 be $f(x)$ and 2 be $h(x)$.
$f(x) = x^2 - 6x + 11$ (given)
By reflecting $f(x)$ along y axis,
$g(x) = f(-x) = x^2 + 6x + 11$
By translating $g(x)$ along x axis for 10 units to the right,
$h(x) = g(x-10) = (x-10)^2 + 6(x-10) + 11 = x^2 - 20x + 100 + 6x - 60 + 11 = x^2 - 14x + 55$ $(5 \le x \le 10)$

They are both correct because both methods can obtain the image of curve 2 from curve 1 by transformation of graphs. The only difference is Ahmed considers to change of points after the reflection, while Delinda considers the positioning of the whole curve for reflection and translation.



To complete the above logo, curve 1 is transformed into curve 3, and curve 2 is transformed into curve 4

16. What single transformation is performed on curves 1 and 2 in order to end up with the logo?

Curve 1 and 2 is reflected along y=6 to form curve 3 and 4.

17. What is the equation of curve 3?

Let equation of curve 1 = f(x) and curve 3 be g(x).
f(x)= $x^2-6x+11$ (given)
g(x)=

End of Assessment